

## Dynamic aggregate supply and demand: a pedagogical application

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### ABSTRACT

In this paper, a simple dynamic aggregate demand and supply model is developed as a useful pedagogical model alongside the usual AD/AS version. Nearly all of the macroeconomic information the public receives is presented in this rate-of-change form. Using US contemporaneous, quarterly data from 1980 through 2018, dynamic aggregate supply and demand functions are theoretically derived and empirically determined using 2SLS regressions. These real-world results are then used to construct a macro simulation model of the US economy in Microsoft Excel, which is provided.

The model is built to be easily used in an undergraduate macroeconomics course. It enables the user to simulate a variety of economic shocks, including changes in the money supply rates of growth, velocity of money rates of growth, anticipated inflation shocks, and wage growth rate shocks. The resulting impacts are realistic, having been based on actual US data. Examples include both a monetary (demand) and wage (supply) shocks as teaching examples. The Great Recession is also simulated using both supply and demand shocks, with the results of the model being compared to actual historical data during the Great Recession, allowing for a more robust in-class example.

Keywords: Dynamic aggregate demand and aggregate supply, teaching macroeconomics, economic data, economic shocks, the US economy, macroeconomic theory

JEL Classifications: A20, A22, E00

## 1. INTRODUCTION

It is common to hear concerns raised from the media about the possibility of overheating economies, which will possibly bring on inflationary pressures; or to encounter puzzlement that inflation is stubbornly difficult to raise; or to see everyone waiting for the Fed to make its pronouncements about monetary policy. There is a relationship between inflation and the economy, but it is not necessarily clear what that relationship is or the strength of the relationship. Undergraduate economics students are often taught some standard aggregate demand-aggregate supply models, or some versions of new Keynesian models to help with this. This paper introduces a tractable dynamic macroeconomic model suitable for undergraduate economics students to help facilitate a better understanding of macroeconomics. In addition, the model is calibrated in a Microsoft Excel file to show the short- and long-run implications of economic shocks. Teaching applications are also provided.

The variables used in the model are purposeful. It is not commonly presented that the price level is 225.8, instead the news reports state that the inflation rate is 1.7%. Likewise, GDP is not typically conveyed as \$21.6 trillion, but, more commonly what is reported is that GDP grew at a 2.4% in the third quarter. It seems reasonable, then, especially since most students who read these books and see these models are non-majors, that the models conform to what everyone is seeing on a regular basis. Unfortunately, most principles of economics and many intermediate macroeconomics textbooks use aggregate demand and aggregate supply as the sole pedagogical tool to show how the macroeconomy responds to fiscal and monetary policies, to show how the macroeconomy equilibrates, to show how it is impacted by economic shocks, and so on. This approach has been a pedagogical staple for many years.

Only recently have intermediate texts tended to bring in the dynamics of inflation and growth rates, although principles texts have still not adopted this pedagogical method. One recent example of this dynamic approach is Mankiw (2016). Another excellent example is found in Wuthisatian and Thanetsunthorn (2019), although it is not yet used in a textbook. These approaches are to be applauded. Still, the model presented here is fundamentally different from these examples. Mankiw, for instance, uses a five-equation dynamic approach and uses Lucas's 'dynamic aggregate supply' (DAS) curve for the supply side of the model. But his DAS curve is essentially a Phillips curve, a standard inventory adjustment equation commonly used in supply and demand models and is not a true aggregate supply function. Moreover, Mankiw's model is too advanced for introductory macroeconomics. Wuthisatian and Thanetsunthorn, on the other hand, use a standard aggregate demand and supply model that incorporates a perfectly inelastic long-run aggregate supply. With their model, they can introduce exogenous shocks and generate response movements back toward a new equilibrium. Their model is fairly sophisticated and well thought out; however, it too is quite advanced and is inappropriate for introductory students.

To meet the delicate requirements that the macro-models use familiar dynamic variables and while still being simple enough for introductory students, a simulation model is presented here. It incorporates dynamics in a way that can be taught at the principles or intermediate level using the provided dynamic aggregate demand (DAD) – dynamic aggregate supply (DAS). The theoretical foundation is suitable for undergraduate students. The instructor is provided an instructional tool to facilitate the application of the dynamic aggregate demand/aggregate supply model into a standard undergraduate macroeconomics classroom. To provide realism, US quarterly data from 1980 to 2018 were used to derive realistic dynamic aggregate demand and aggregate supply functions by two-stage least squares (2SLS) regression analysis. Curves

representing these functions are presented in Microsoft Excel with real GDP growth rates on the horizontal axis and inflation on the vertical axis. These curves represent the long-run average positions for the DAD and DAS curves over the 38-year period.

The Excel simulation model is designed to equilibrate at the long-run average positions for real GDP growth rates and inflation over the past 38 years. Exogenous shocks can be introduced, automatically generating response functions showing the inexorable march back toward long-run equilibrium. Specifically, the instructor can illustrate both the short- and long-run inflation and real GDP growth rate effects of exogenously generated one-time, or permanent changes in the rates of growth of the money supply, velocity of money, anticipated inflationary expectations, and nominal wage shocks. For instance, it is simple for the instructor to cover demand management policies. Fiscal policies shocks (government spending, taxes, etc.) alter the velocity of money variable in the model, whereas monetary policy shocks are seen through the money growth rate variable. By simply inputting a positive or negative shock value into the Excel simulation model, the new long-run equilibrium can be determined for a permanent shock, or the dynamic short-term fluctuations can be seen from a temporary dampening shock. This simplicity of use allows for easy incorporation into the classroom. Examples are provided for a one-time monetary shock, a one-time wage shock, and a simultaneous set of one-time shocks to mimic the actual behavior of inflation and real GDP growth rates during this century's Great Recession. This tool is a primary contribution of this paper and highly useful for teachers of economics.

As an important caveat note that, although the quantitative exposition is based on historical US data and the model generates actual numbers, those numbers are not meant to be interpreted as exact figures or projections for the US economy. They should be interpreted only as semi-realistic approximations of US data, suitable for undergraduate consumption. The accuracy of the data and omitted variable bias are only a few of the factors that may contribute to potential inaccuracies in the specific numbers used and generated in the model developed here. However, the model serves as a benchmark for the analysis sufficient to give students a reasonable feel for how the economy operates and responds to shocks.

Most of the current literature on the relationship between inflation and GDP growth rates stems from Barro (1996), who found that, world-wide, inflation is negatively correlated with growth rates. Many of the countries in his data set had relatively high inflation rates compared to recent US experience and thus researchers queried whether his result was robust or instead was sensitive only to high inflation rates. That question touched off a series of papers searching for turning point rates, or thresholds, of inflation which might still generate the Phillips-like positive correlations. Most of these studies analyze one or a small number of countries. Some of those inquiries found supporting negative correlations, such as Bruno and Easterly (1996), Andrés and Hernando (1999), Khan and Senhadji (2001), Gokal and Hanif (2004), and Ahmed and Mortaza (2010). But others found either no, or positive correlations between growth and inflation, in direct contrast to Barro's findings. For instance, Sarel (1996), Mallik and Chowdhury (2001), Pollin and Zhu (2006), and Datta and Mukhopadyay (2011), at least at some threshold and all found positive correlations between growth and inflation. It is evident that the question is still unsettled.

The paper will proceed as follows. Section 2 will provide the theoretical foundation of the model and utilize US quarterly data to numerically solve the equilibrium. Section 3 will outline the supplemental Microsoft Excel file and provide examples of the short- and long-run implications of different economic shocks. Section 4 concludes.

## 2. THE MODEL

### 2.1 The Dynamic Aggregate Supply and Demand Curves

The model uses the following endogenous variables: inflation, real GDP growth rate, money supply growth rate, velocity growth rate, nominal wage growth rate, and anticipated inflation, and incorporates dynamic aggregate supply or demand shocks through exogenous changes in wage growth, money growth, velocity growth, and anticipated inflation. These exogenous shocks represent the impacts of many other variables on the economy. For instance, velocity shocks can come from changes in government spending, changes in taxes, changes in world trade, etc. anticipated inflation shocks can come from concerns about a recession, a new government administration, implementation of a higher minimum wage, etc. The instructor has ample opportunity to discuss how exogenous variables can impact the economy, and then watch how it impacts it in the simulation model.

To illustrate where the DAS curve comes from, a simple aggregate supply model is developed below incorporating a linear production function and a labor market. The production function's real GDP,  $Y$ , is positively influenced by the amount of labor,  $N$ . The labor wage supply curve is positively influenced by labor, price level,  $P$ , and expected price level,  $P^e$ . Labor wage demand is negatively related to labor and positively related to the price level. The final equation is the equilibrium condition.

$$(1) \quad Y = \alpha_0 + \alpha_1 N$$

$$(2) \quad w^s = w_0 + \beta_0 N + \beta_1 P + \beta_2 P^e$$

$$(3) \quad w^d = w_1 - \gamma_0 \alpha_1 N + \gamma_1 P$$

$$(4) \quad w^s = w^d$$

Equation (5) is the dynamic aggregate supply curve found by taking logs and multiplying through by a time derivative of the price level function derived from (1) – (4) above. Appendix A provides the derivation of the dynamic aggregate supply curve.

$$(5) \quad \dot{P} = \delta_0 + \delta_1 \dot{P}^e + \delta_2 \dot{Y}$$

The dynamic aggregate supply equation shows that price inflation is a function of labor market movements and reactions, expected price level inflation, and GDP growth rate respectively. The dot over the variable denotes a percentage rate of growth. Using US quarterly data from 1980 until 2018, Table A.1 in Appendix C provides an initial empirical analysis of the dynamic aggregate supply equation using wage growth as a proxy for labor market machinations. The results in column 1 (Standard OLS) of Table A.1 are of the expected sign and each variable is statistically significant. However, this model does not yet incorporate the aggregate demand side of the economy. To obtain the aggregate demand side of the model, the dynamic equation of exchange is introduced, which is derived and linearized by taking the logs of the famous

equation of exchange and multiplying through by the time derivatives to obtain equation (6).

$$(6) \quad \dot{M} + \dot{V} \cong \dot{P} + \dot{Y}$$

This dynamic equation of exchange can be used to fully account for simultaneous changes in inflation and GDP growth rates caused by dynamic aggregate demand movements, when finding the dynamic aggregate supply curve. Table A.1 in Appendix C presents a second empirical result utilizing a two stage least squares technique. This technique allows us to identify which of the curves was moving in order to generate the rate of inflation or GDP rate of growth. The statistical results are as expected, but the instructor can point out to the students how different the coefficients are after accounting for, and controlling for, simultaneous background movements of the DAD. The results in the 2SLS column will serve as the baseline for the simulation model presented below.

Anticipated price shocks can easily be incorporated into this analysis and will provide a useful tool when looking at different shocks to the economy. The accompanying Microsoft Excel file incorporates this possible shock. For example, an anticipated price shock may include crude oil price inflation as one more component proxy of dynamic aggregate supply. These results are available upon request, but do not change the fundamental results.

Using the same 2SLS technique to solve for the dynamic aggregate demand curve, which again is the rate of change form of the equation of exchange, finds statistically significant signs for the variables as expected. As was the case with finding the DAS curve, the simultaneous movements of the DAS curve must be accounted for and controlled for in order to obtain the proper DAD. The results are presented in Table A.2 in Appendix C. The coefficient of determination is diminished compared to the DAS curve, which is surprising given that the equation of exchange is an identity (although the dynamic form, DAD, is not). From a theoretical point of view, the expected results would be a '-1' coefficient for the real GDP growth rate and unity for both the M2 rate of growth and the M2 velocity rate of growth. The actual results are close to the values but not exact. The differences can be attributed to many factors including the use of an intercept, the measurement of GDP and the measurement of the CPI. This empirical exercise services as a useful approximation of the model.

## 2.2 Reduced Form Equilibrium

The dynamic equation of exchange (equation 6) is a binding constraint. Solving that equation for real GDP rate of growth gives

$$(7) \quad \dot{Y} \cong \dot{M} + \dot{V} - \dot{P}$$

Subbing (7) into (5) generates the reduced-form equilibrium inflation function (8) in terms of wage rate of growth, inflation expectations, money rate of growth, and velocity rate of growth.

$$(8) \quad \dot{P} = \frac{1}{(1+\delta_2)} [\delta_0 + \delta_1 \dot{P}^e + \delta_2 (\dot{M} + \dot{V})]$$

Subbing (7) into (5) also generates the reduced-form real GDP growth rate function (9) in terms of the same variables as in equation (8): wage rate of growth, inflation expectations, money rate of growth, and velocity rate of growth.

$$(9) \quad \dot{Y} = \frac{1}{(1+\delta_2)} [\dot{M} + \dot{V} - (\delta_0 + \delta_1 \dot{P}^e)]$$

Based on equation (8), inflation should be impacted equally by the rates of growth of money supply or velocity, i.e. both  $\frac{\partial \dot{P}}{\partial \dot{M}}$  and  $\frac{\partial \dot{P}}{\partial \dot{V}}$  coefficients are  $\frac{\delta_2}{(1+\delta_2)} < 1 = \varphi$ . Moreover, the final GDP rate of growth must be subsumed under the remaining variables, including expected inflation, which are all supply-side variables and largely negative.

When the preliminary regression was run,  $\varphi$  turned out to be roughly 0.19 and was, as expected, approximately 0.19 for both money and velocity rates of growth. Using 0.19 to solve for  $\delta_2$  above  $\delta_2$  should equal 0.234. That is, empirically, with everything held constant, the expectation is

$$\delta_2 = \frac{\partial \dot{P}}{\partial \dot{Y}} \cong 0.234$$

where  $\delta_2$  is the slope of the dynamic aggregate supply curve. This is empirically testable as a part of the 2SLS regression. As the real rate of GDP rises,  $\delta_2$  implies that inflation will rise by roughly a quarter of the rate of growth of GDP, everything else accounted for. If GDP increases its rate of growth by a percent, there will be a small corresponding increase of inflation by 0.23% - again, all else held constant. Regression analysis using simple OLS supports the theoretical finding above. The empirical representation of the dynamic aggregate supply curve shows all statistically significant variables with the correct theoretical comparative static signs. The slope, however, is 0.10 rather than the expected 0.23. This is likely due to missing variables or the fact that the 2SLS technique was not used. Using a 2SLS approach, the slope is closer to the predicted amount at 0.189 and the resulting dynamic aggregate supply equation has a very high adjusted R-square.

The DAD – DAS equations are as follows based on the regression results presented in Appendix B. The values reflect the US economy between 1980 and 2018 which encompasses 153 quarterly periods.

$$\text{DAS: } \dot{P} = -2.814 + 1.285\dot{P}^e + 0.233\dot{W} + 0.189\dot{Y}$$

$$\text{DAD: } \dot{P} = -0.249 + 0.978\dot{M} + 1.046\dot{V} - 0.884\dot{Y}$$

Based on US quarterly data from 1980 – 2018, the average anticipated inflation rate is 3.363, the average wage growth rate is 3.416, the average M2 growth rate is 6.126 and the average M2 velocity growth rate is -0.611. These are all in percentage form. Plugging in those averages simplifies the DAS and DAD curves to be

$$\text{DAS: } \dot{P} = 2.3034 + 0.189\dot{Y}$$

$$\text{DAD: } \dot{P} = 5.1031 - 0.884\dot{Y}$$



Figure 1 (Appendix A) graphically shows the long-run dynamic aggregate demand and aggregate supply curves based on the US quarterly data between 1980 and 2018. The curves intersect at long-run equilibrium at an inflation rate of 2.796 and a real GDP growth rate of 2.613. It is important to note that, while the long-run averages are exact, the curves themselves cannot be interpreted as exact, but only as approximations. The accuracy of the data and omitted variable bias are only a few of the factors that may contribute to potential inaccuracies in the specific numbers. However, these numbers do serve as a strong benchmark for the analysis.

### 3. TEACHING APPLICATION – ECONOMIC SHOCKS

A corresponding Microsoft Excel file is included to facilitate the use in the classroom. The file contains three tabs. The first is entitled ‘Underlying Data – FRED.’ In this tab, the data are provided and organized on a quarterly basis from Q1 1980 to Q4 2018. The data are also available from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. The second tab, entitled ‘DAD – DAS Curves,’ derives Figure 1 (Appendix A). No changes to the first two tabs are needed for the simulations of economic shocks.

The third, and final, tab is entitled ‘Simulation.’ Appendix D details the equations that were used, typed into the cells, in the Excel simulation model to generate dynamics. The equations are based on the regression results using the same intercepts and slopes as well as the same coefficients for the explanatory variables. The additional aspect, to make it dynamic, allows for separate or simultaneous shocks to be applied, is a simple method of dampening the shocks. A shock is applied and loses strength as it dissipates over time. The dampening process used for the model is  $x \text{ shock} = 0.8 * x \text{ shock}_{t-1}$  for the persistence of shock to any variable  $x$ . The shock then predictably loses strength each period so that it is only 80% of what it was the period before. For instance, an original shock of +2 (percent rate of growth) becomes: 1.6, 1.28, 1.024, 0.8192, etc. period to period as it fades toward zero. This simple method mimics actual behaviors surprisingly well.

To simulate the shocks, only cells M6, N6, O6, and/or P6 of the ‘Simulation’ tab need to be changed. The dynamics of each variable will automatically adjust given the persistence described above. The corresponding figure will adjust as well.

A shock to the model can be interpreted in two ways. First, the initial period responses represent the full shifts of DAD or DAS curves away from the original positions in Figure 1. This would also be the final new equilibrium if the shock was once-for-all permanent. However, if the shock is transitory the movement back to equilibrium is governed by the dampening process and is viewed graphically as response functions. Examples are given below.

The model is subjected to a series of economic shocks. M2 growth rates and M2 velocity growth rates are used to shock the dynamic aggregate demand curve and anticipated inflation rates and wage growth rates are used to shock the dynamic aggregate supply curve. The initial equilibrium has an inflation rate of 2.8 and a GDP growth rate of 2.6. These results are quantitatively very close to the values obtained above, with the only discrepancy being rounding errors. Appendix E provides a screenshot of how the shocks are incorporated into Excel. For example, Figure A.1 in Appendix E shows the simulation without any shocks (all shocks set to 0).

The shocks below are temporary and damped over time. For example, a 1 percent shock to the money supply growth rate is dissipated by 20% each period. Over 46 simulated periods, a 1 percent shock will be reduced to 0.00003484, essentially returning the pre-shock equilibrium.

The following applications consist of a 1 percent increase in the monetary growth— an aggregate demand shock; a 1 percent decrease in the wage growth rate — an aggregate supply shock; and simulation of the “Great Recession” in which each of the four factors are simultaneously shocked.

### 3.1 Positive Monetary Shock

The first application is to temporarily increase the rate of growth of the money supply by 1 percent. As with each of the applications, the shock has a persistence of 0.8 per period. Using 1980 to 2018 quarterly US data, the average growth rate of the M2 money supply was 6.126. This shock temporarily increases the growth of money supply to 7.126. Figure A.2 in Appendix E shows how the shock is applied in the corresponding Excel file under the ‘Simulation’ tab by adding ‘+1’ into cell M6.

Figure 2 (Appendix A) shows the initial and transitional dynamics of the positive shock to the aggregate demand. This expansionary policy change increases both inflation rate and GDP growth rates. Specifically, inflation increases from 2.802 to 2.975 in the short run and the GDP growth rates increase from 2.601 to 3.513. The short-run change in the GDP growth rate is quantitatively greater than the shock to the inflation rate. This is due to the relatively flat dynamic aggregate supply curve and also the *ceteris paribus* nature of the shock. Over time, the shock dissipates and the economy returns to the initial equilibrium.

### 3.2 Negative Wage Growth Shock

The second application is to consider a negative shock to the wage growth rate. Over the period between 1980 and 2018, the average wage growth rate in the US was 3.416. The shock temporarily lowers the wage growth rate to 2.416, again dissipating over time. After 46 simulated periods, the shock is diminished, again, to 0.00003484, and the economy returns to the previous long run equilibrium. Figure A.3 in Appendix E shows how the shock is applied in the corresponding Excel file under the ‘Simulation’ tab by adding ‘-1’ into cell P6.

Figure 3 (Appendix A) shows that in the short run, this negative shock to the wage growth rate is expansionary in nature, shifting the dynamic aggregate supply curve to the right, increasing the GDP growth rate for a given rate of inflation. Firms have lower employee costs in the short run. GDP growth increases from 2.601 to 2.814 in the short run, while the inflation rate drops from 2.802 to 2.615.

### 3.3 A Simulation of the Great Recession

The model can be used to see how deep recessions slowly recover. In the late 2000s the US had one of its worst economic periods in history. That period, now known as the Great Recession, is usually considered the time period between late 2007 until the middle of 2009, but the effects of the recession lingered much longer.



Figure 4a (Appendix A) shows how the actual inflation and real GDP behaved just before, during, and after the Great Recession. The chosen dates are from first quarter 2007 to the third quarter 2012 for this illustration. As is easily seen, the economy was falling at roughly a 4% rate at the nadir and remained growing negatively for more than a year and a half. That is an extraordinarily deep recession.

Using the simulation model, simultaneous shocks to all of the shock variables is used to mimic the Great Recession. The shock levels were based on the actual shocks that occurred at the time, but they are not exactly the same. The simulation model shocks all of the variables at the same time, whereas the reality is that some of the shocks occur before or after others because of reactions or learning, etc. Still, the model does a robust job of duplicating the extraordinary behaviors actually observed.

To mimic the Great Recession shocks were imposed on the model simultaneously. The shocks were: M2 rate of growth = 1%, M2V rate of growth = -11%, expected inflation = -2%, and nominal wage rate of growth = -2%. Figure A.4 in Appendix C shows how the shocks are applied in the corresponding Excel file under the 'Simulation' tab by adding '+1' into cell M6, a '-11' into cell N6, a '-2' into cell O2, and a '-2' into cell P2. Again, as explained earlier, these are not exactly what occurred, but do give a reasonable portrait of what happened. For instance, M2V rate of growth did indeed fall by around 11 percent at the outset of the recession and was near that for 3 periods before moving back more toward normal. The simple simulation here applies the -11% shock and then lets it persist with a frictional drag, each next period's shock being 80% of the previous period. The structured dampening shock is, of course, not exactly how it happened in reality. Even with this limitation, the model does a nice job of showing the severity and length of the Great Recession.

Figure 4b (Appendix A) provides the simulated results. Overall, the simulated model in Figure 4b matches the actual data in Figure 4a surprisingly well.

#### 4. SUMMARY AND CONCLUSION

Nearly all of the macroeconomic information the public receives is in a rate-of-change form, i.e. in dynamic form. This should be the case in the economics classroom as well. In this paper a simple long-run dynamic aggregate demand and supply model is promoted as a better pedagogical model than the usual static AD/AS version. Using US contemporaneous, quarterly data from 1980 through 2018, the dynamic aggregate supply and demand functions are estimated using 2SLS regression techniques. Using those results a dynamic macroeconomic simulation model is built in Excel that allows students in an undergraduate macroeconomics course to easily simulate how the economy looks and how it reacts to shocks.

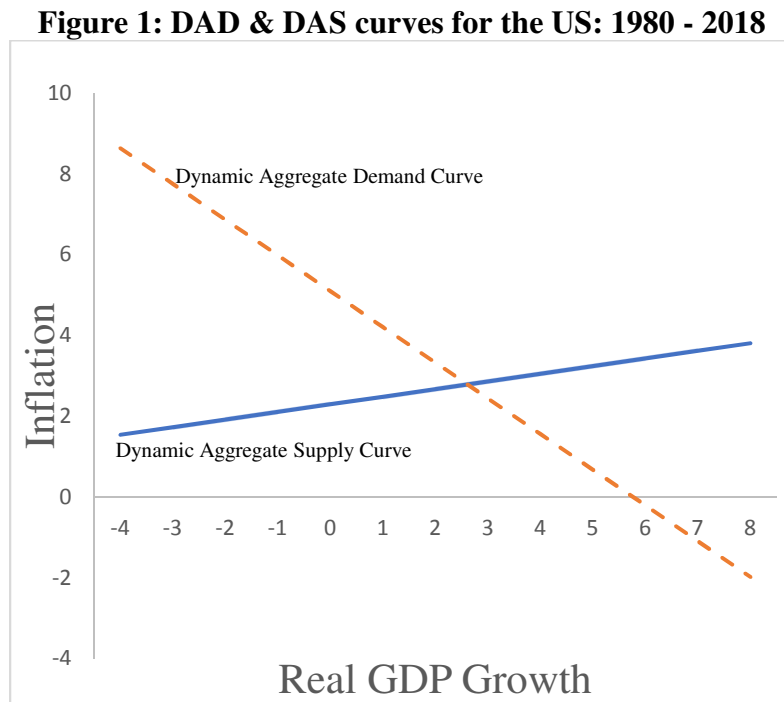
The simulation model provided here is a simple-to-use learning tool that gives the student an accurate sense of how the economy behaves. The model is robust in allowing the user to simulate dynamic aggregate demand and supply shocks to the economy, including changes in the rates of growth of the money supply, velocity of money, anticipated price level, and wages. For illustration, both a monetary (demand) and wage (supply) shock were analyzed as teaching examples. Screenshots of these are provided, depicted in both graphical and response function forms. For a more robust, realistic in-class example the Great Recession is simulated using a combination of simultaneous supply and demand shocks. The response function results were compared to historical data during the Great Recession, showing a surprisingly good match between the two.

The model also allows the instructor to examine a wide variety of the macroeconomic ideas. For example, fully anticipated government spending shocks (use equal and simultaneous shocks to velocity and anticipated inflation); over- or under-anticipated monetary shocks (use simultaneous, but unequal shocks to money growth and anticipated inflation); a permanent, but fully anticipated minimum wage increase (use equal and simultaneous shocks to wage growth and anticipated inflation); etc.

Using rates-of-change forms of macroeconomic variables that correspond to what students see outside the classroom, along with analyzing macroeconomic behaviors using the corresponding simulation model, brings a superior pedagogical approach to learning macroeconomics.

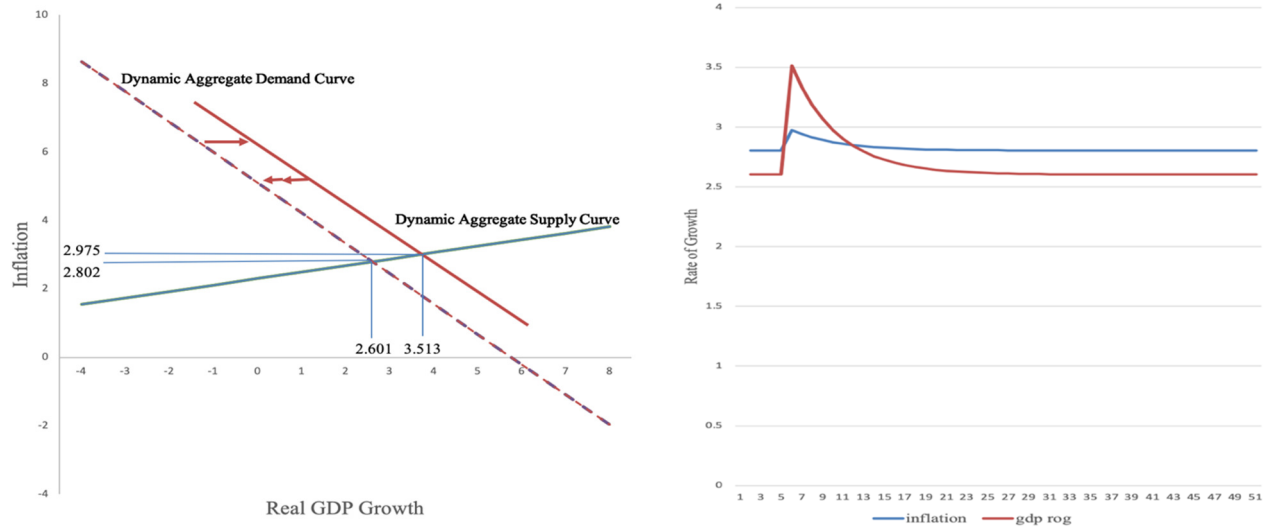
## References

- Ahmed, S., & Mortaza, M. G. (2010). *Inflation and economic growth in Bangladesh: 1981-2005* (No. id: 3033).
- Andrés, J., & Hernando, I. (1999). Does inflation harm economic growth? Evidence from the OECD. In *The costs and benefits of price stability* (pp. 315-348). University of Chicago Press.
- Barro, R. J. (1996). *Determinants of economic growth: A cross-country empirical study* (No. w5698). National Bureau of Economic Research.
- Bruno, M., & Easterly, W. (1996). Inflation and growth: in search of a stable relationship. *Federal Reserve Bank of St. Louis Review*, 78(May/June 1996).
- Datta, K., & Mukhopadhyay, C. K. (2011). Relationship between inflation and economic growth in Malaysia-an econometric review. In *International Conference on Economics and Finance Research* (Vol. 4, No. 1, pp. 415-419).
- Gokal, V., & Hanif, S. (2004). *Relationship between inflation and economic growth*. Economics Department, Reserve Bank of Fiji.
- Mallik, G., & Chowdhury, A. (2001). Inflation and economic growth: evidence from four south Asian countries. *Asia-Pacific Development Journal*, 8(1), 123-135.
- Mankiw, G. (2016). *Macroeconomics*. Worth Publishers, New York, NY. 9<sup>th</sup> Edition.
- Federal Reserve Bank of St. Louis, Economic Research, FRED Economic Data (2019).
- Khan, M. S., & Senhadji, A. S. (2001). Threshold effects in the relationship between inflation and growth. *IMF Staff papers*, 48(1), 1-21.
- Pollin, R., & Zhu, A. (2006). Inflation and economic growth: A cross-country nonlinear analysis. *Journal of post Keynesian economics*, 28(4), 593-614.
- Sarel, M. (1996). Nonlinear effects of inflation on economic growth. *Staff Papers*, 43(1), 199-215.
- Wuthisatian, R., & Thanetsunthorn, N. (2019). Teaching macroeconomics with data: Materials for enhancing students' quantitative skills. *International Review of Economics Education*, 30.

**Appendix A: Derivation of the Long-Run DAS curve**

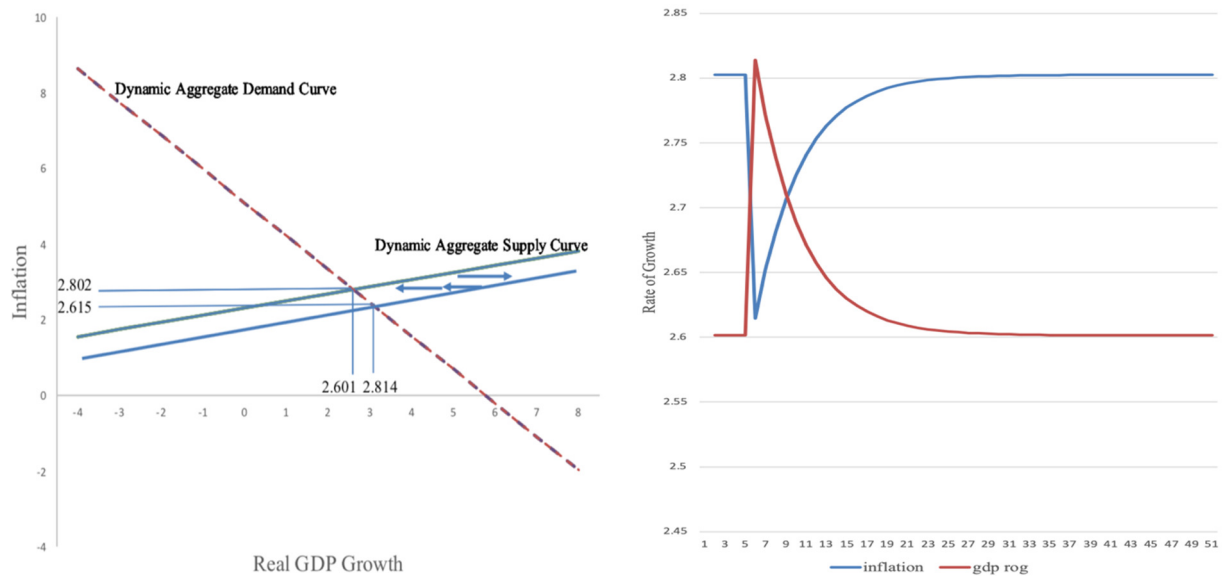
Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis.

**Figure 2: Positive Monetary Shock**



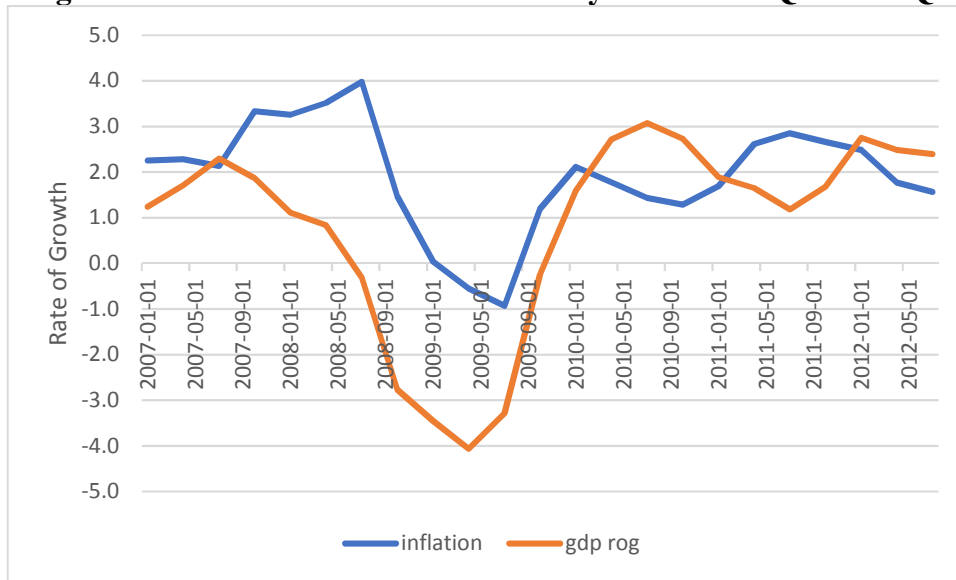
Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data are obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. The left diagram is the dynamic aggregate supply (DAS) – dynamic aggregate demand model (DAD) and the right diagram is the simulated response functions over 50 periods with the shock occurring in period 4.

**Figure 3: Negative Wage Shock**



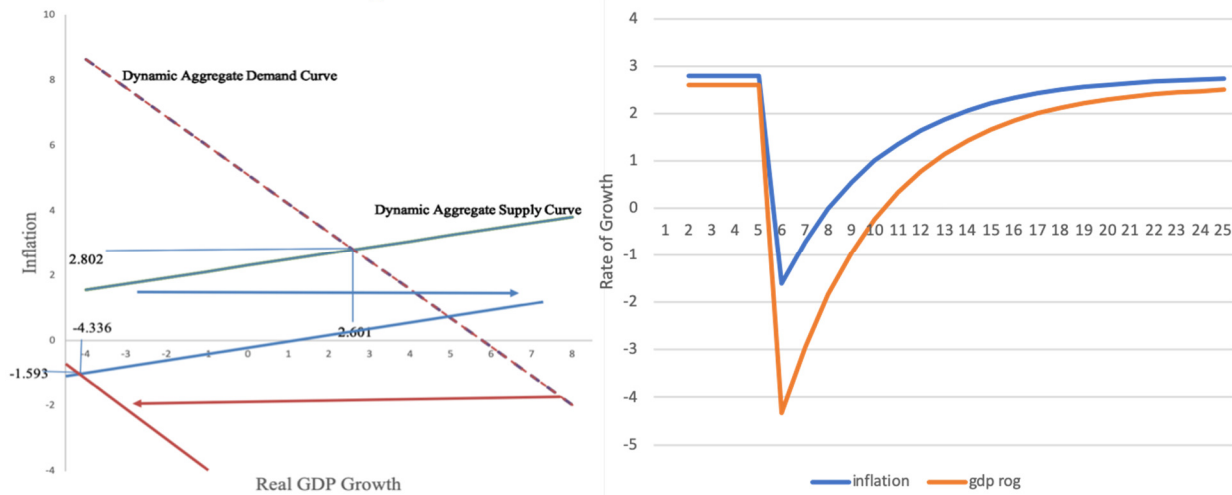
Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data are obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. The left diagram is the dynamic aggregate supply (DAS) – dynamic aggregate demand model (DAD) and the right diagram is the simulated response functions over 50 periods with the shock occurring in period 4.

**Figure 4a: The Great Recession Actual Dynamics 2007Q1 – 2012Q3**



Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data are obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. The dates range from Q1 2007 to Q3 2012.

**Figure 4b: The Great Recession Simulated**



Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data are obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. The left diagram is the dynamic aggregate supply (DAS) – dynamic aggregate demand model (DAD) and the right diagram is the simulated response functions over 50 periods with the shock occurring in period 4.

**Appendix B: Derivation of the Long-Run DAS curve**

$$Y = \alpha_0 + \alpha_1 N$$

$$w^s = w_0 + \beta_0 N + \beta_1 P + \beta_2 P^e$$

$$w^d = w_1 - \gamma_0 a_1 N + \gamma_1 P$$

$$w^s = w^d$$

$$w_0 + \beta_0 N + \beta_1 P + \beta_2 P^e = w_1 - \gamma_0 N + \gamma_1 P$$

$$(\gamma_0 a_1 + \beta_0) N = w_1 - w_0 + (\gamma_1 - \beta_1) P - \beta_2 P^e$$

$$N = \left[ \frac{w_1 - w_0 + (\gamma_1 - \beta_1) P - \beta_2 P^e}{(\gamma_0 a_1 + \beta_0)} \right]$$

Plug this into equation (1) gives

$$Y = \alpha_0 + \alpha_1 \left[ \frac{w_1 - w_0 + (\gamma_1 - \beta_1) P - \beta_2 P^e}{(\gamma_0 a_1 + \beta_0)} \right]$$

Solving for P leaves us with

$$P = \left[ \left( \frac{w_1 - w_0}{\gamma_1 - \beta_1} \right) + \left( \frac{\gamma_0 a_1 + \beta_0}{(\gamma_1 - \beta_1) \alpha_1} \right) \cdot \left\{ -\alpha_0 + \left( \frac{\beta_2 \alpha_1}{\gamma_0 a_1 + \beta_0} \right) P^e \right\} \right] + \left( \frac{\gamma_0 a_1 + \beta_0}{(\gamma_1 - \beta_1) \alpha_1} \right) Y$$

Which simplifies to

$$P = (\delta_0 + \delta_1 P^e) + \delta_2 Y$$

Putting in rates of change form by taking the log derivatives gives

$$\dot{P} = \delta_0 + \delta_1 \dot{P}^e + \delta_2 \dot{Y}$$



### Appendix C: Regression Results

Table A.1 shows the regression results for US quarterly data from 1980 to 2018 – 153 quarterly periods based on the dynamic aggregate supply curve. The dependent variable is the rate of inflation and the independent variables are the expected inflation rate measured by the University of Michigan Survey (MICH) from the FRED economic dataset, the nominal wage growth and the real GDP growth.

**Table A.1: Dynamic Aggregate Supply**

|                            | Standard OLS             | 2SLS                     |
|----------------------------|--------------------------|--------------------------|
| <b>Intercept</b>           | -2.4992***<br>(-12.2741) | -2.8138***<br>(-67.8886) |
| <b>Expected Inflation</b>  | 1.2516***<br>(20.1555)   | 1.2849***<br>(101.6511)  |
| <b>Nominal Wage Growth</b> | 0.2388***<br>(5.4779)    | 0.2329***<br>(26.2447)   |
| <b>Real GDP Growth</b>     | 0.1035***<br>(3.3635)    | 0.1888***<br>(30.1376)   |
| <b>Observations</b>        | 153                      | 153                      |
| <b>R-square</b>            | 0.866                    | 0.994                    |
| <b>F-statistic</b>         | 329.566                  | 8165.861                 |

Notes: \*\*\* denotes significance at the 1% level; \*\* at the 5% level; and \* at the 10% level.

As expected, the results in Table A.1 are of the correct sign and highly significant. The 2SLS technique allows us to identify which of the curves was moving in order to generate the rate of inflation or the GDP rate of growth. These coefficients are used in the Excel spreadsheet to approximate the US economy.

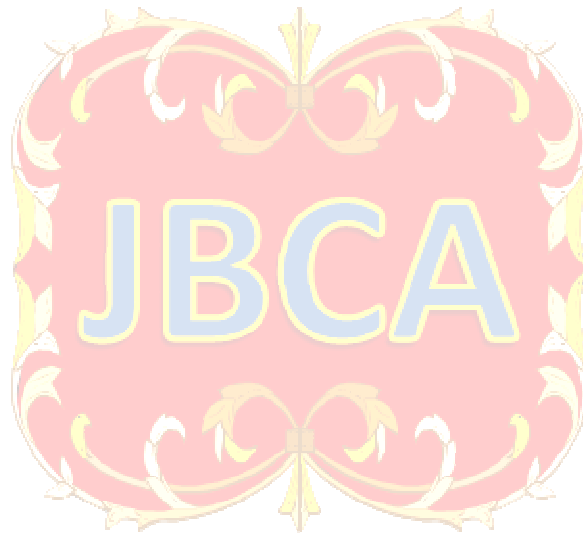
Table A.2 presents the empirical results for the dynamic aggregate demand based on equation (6). From a theoretical point of view, the expected results would be a '-1' coefficient for the real GDP growth rate and unity for both the M2 rate of growth and the M2 velocity rate of growth. The actual results are close to the values but not exact. The differences can be attributed to many factors including the use of an intercept, the measurement of GDP and the measurement of the CPI. This empirical exercise serves as a useful approximation of the model.

**Table A.2: Dynamic Aggregate Demand**

|                                   | 2SLS                     |
|-----------------------------------|--------------------------|
| <b>Intercept</b>                  | -0.2497<br>(-1.2246)     |
| <b>Real GDP Growth</b>            | -0.8840***<br>(-19.9081) |
| <b>M2 Rate of Growth</b>          | 0.9785***<br>(25.9505)   |
| <b>M2 Velocity Rate of Growth</b> | 1.0460***<br>(28.3216)   |

|                     |         |
|---------------------|---------|
|                     |         |
| <b>Observations</b> | 153     |
| <b>R-square</b>     | 0.848   |
| <b>F-statistic</b>  | 283.991 |

Notes: \*\*\* denotes significance at the 1% level; \*\* at the 5% level; and \* at the 10% level.



**Appendix D: Microsoft Excel Macroeconomic Simulation Model**

$$\text{DAD intercept} = 0.978(\text{M2 rog}) + 1.046(\text{V2 rog}) - 0.249$$

$$\text{DAS intercept} = 1.314(\text{expected inflation}) + 0.228(\text{wage rog}) - 2.814$$

$$\text{DAD slope} = \overline{-.884}$$

$$\text{DAS slope} = \overline{.189}$$

$$\text{inflation} = \text{DAD intercept} + \text{DAD slope} \left[ \frac{\text{DAD intercept} - \text{DAS intercept}}{\text{DAS slope} - \text{DAD slope}} \right]$$

$$\text{real gdp rog} = \left[ \frac{\text{DAD intercept} - \text{DAS intercept}}{\text{DAS slope} - \text{DAD slope}} \right]$$

$$\text{M2 rog} = 6.126 + \text{M2 shock}$$

$$\text{V2 rog} = -.611 + \text{V2 shock}$$

$$\text{expected inflation} = 3.363 + \text{expected inflation shock}$$

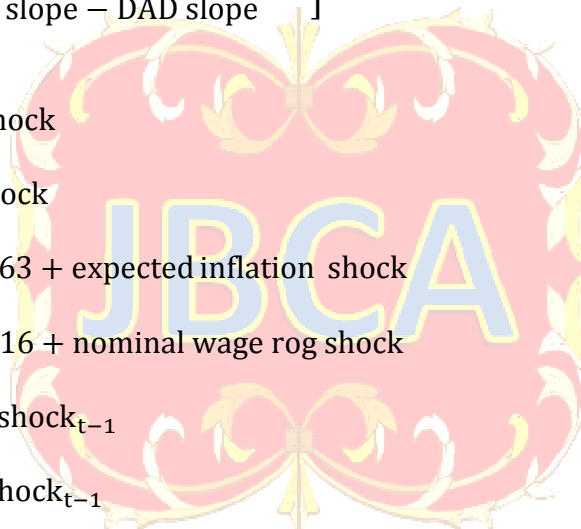
$$\text{nominal wage rog} = 3.416 + \text{nominal wage rog shock}$$

$$\text{M2 shock} = 0; 0.8 * \text{M2 shock}_{t-1}$$

$$\text{V2 shock} = 0; 0.8 * \text{V2 shock}_{t-1}$$

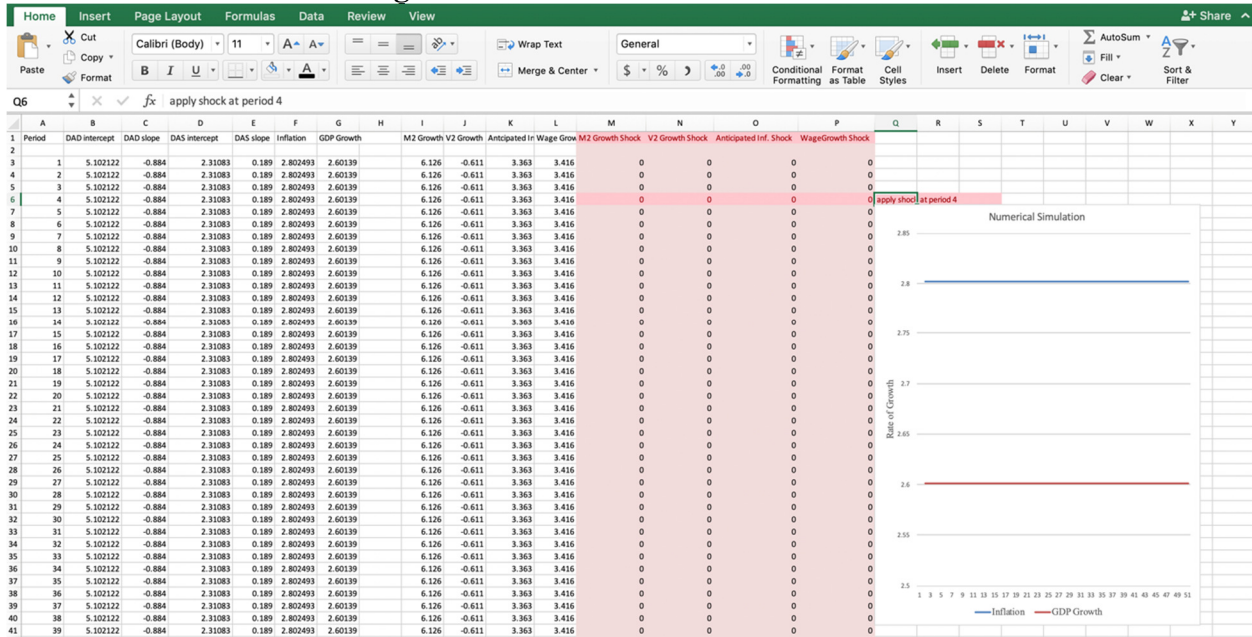
$$\text{expected inflation shock} = 0; 0.8 * \text{expected inflation shock}_{t-1}$$

$$\text{nominal wage rog shock} = 0; 0.8 * \text{nominal wage rog shock}_{t-1}$$



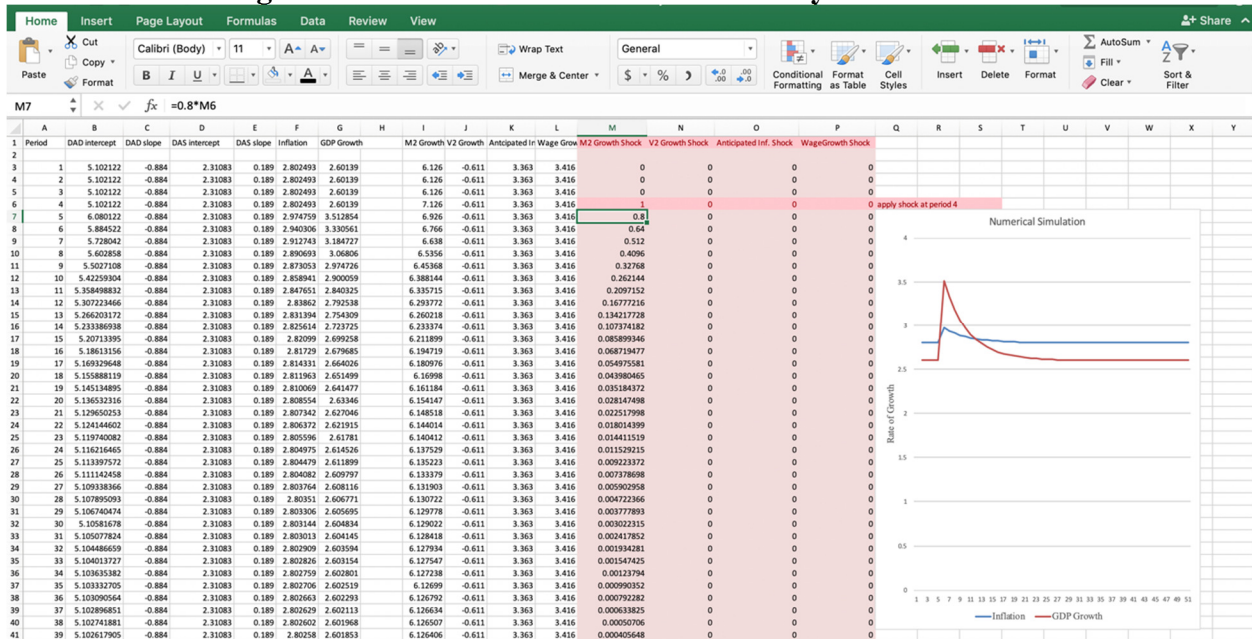
Appendix E: Microsoft Excel Screenshots

Figure A.1: Simulation without Shocks



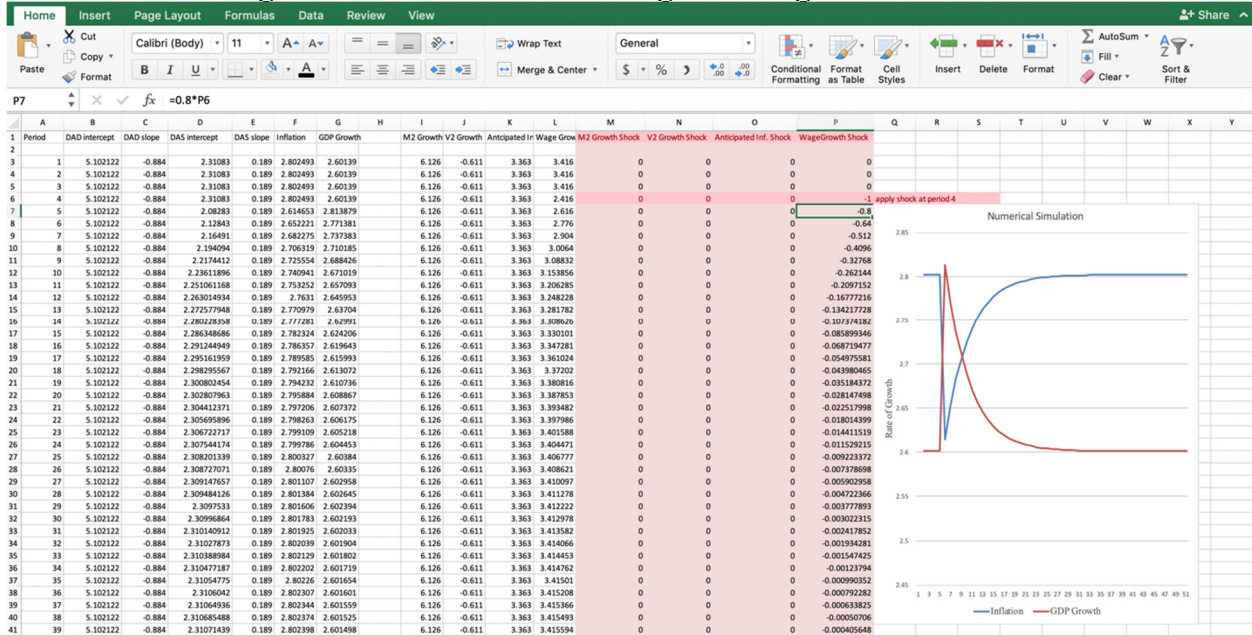
Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. All shocks for money supply growth, velocity of money growth, wage growth, and expected price shocks. The figure above is the baseline with each shock set equal to zero.

Figure A.2: Simulation with Positive Money Growth Shock



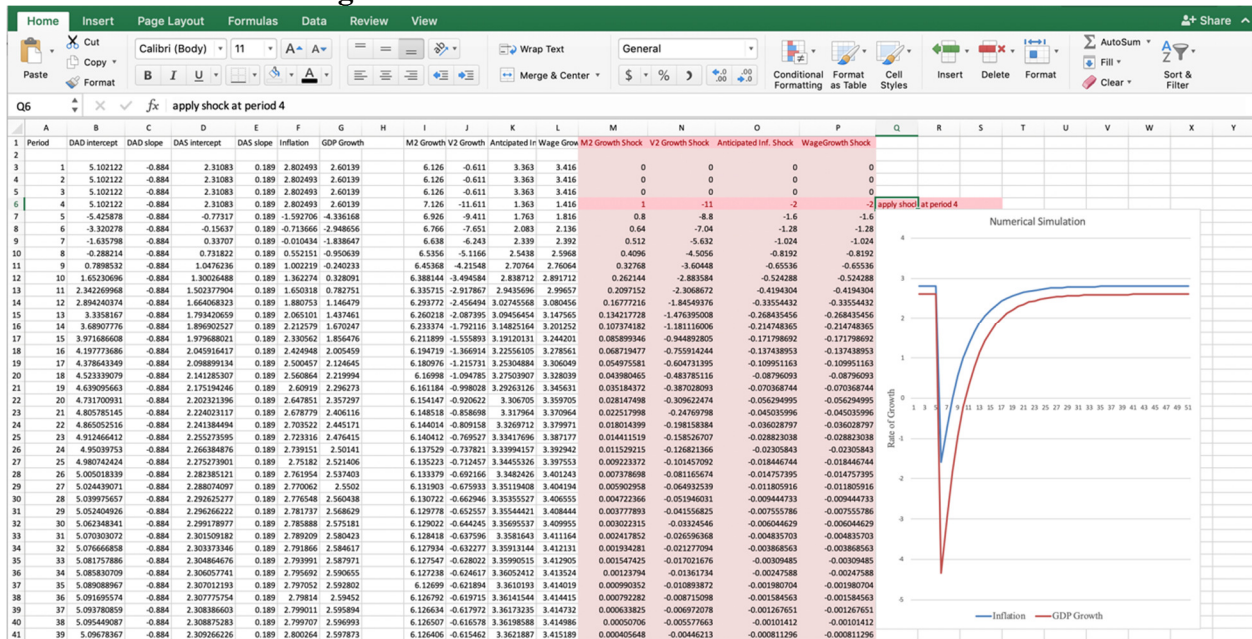
Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. All shocks for money supply growth, velocity of money growth, wage growth, and expected price shocks. The figure applies a shock of +1 to the money growth rate.

Figure A.3: Simulation with Negative Wage Growth Shock



Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. All shocks for money supply growth, velocity of money growth, wage growth, and expected price shocks. The figure applies a shock of -1 to the wage growth rate.

Figure A.4: Simulation for the Great Recession



Notes: US quarterly data from Q1 of 1980 to Q4 of 2018. All data is obtained from the Federal Reserve Economic Data (FRED) provided by the Federal Reserve Bank of St. Louis. All shocks for money supply growth, velocity of money growth, wage growth, and expected price shocks. The figure applies a shock of +1 to the money growth rate, -11 to the velocity growth rate, -2 to the wage growth rate, and -2 to expected inflation.